EXHIBIT C

IN THE UNITED STATES DISTRICT COURT
FOR THE SOUTHERN DISTRICT OF OHIO
WESTERN DIVISION (DAYTON)

PLAYTEX PRODUCTS, INC.,	:	CASE NO. C-1-02-391
Plaintiff,	:	(Judge Thomas M. Rose)
v.	:	
THE PROCTER & GAMBLE DISTRIBUTING COMPANY, et al.	:	SUPPLEMENTAL REPORT OF JAMES C. MOLLER, Ph.D, P.E.
Defendants.	:	

This report responds to the Supplemental Expert Reports of Mario Turchi and Evan Hutchison, which Plaintiff Playtex Products, Inc. ("Playtex") served after issuance of the Court's June 4, 2003 Memorandum Opinion, in which the Court adopted my construction of the term "substantially flattened surfaces." The Court ruled at pp. 10-11 of the Memorandum Opinion that the term "two diametrically opposed, substantially flattened surfaces" means "two opposite opposed surfaces that are flat within a geometric, manufacturing tolerance; the flat surface may or may not have imperfections or surface features such as ribs or treads." In fact, the court made plain (Memorandum Opinion, p. 12) that its interpretation of the phrase "substantially flattened surfaces" is one that "excludes curvature."

Despite rejection of their construction of the term "substantially flattened surfaces," Messrs. Turchi and Hutchison in their supplemental reports assert that the Pearl Plastic nevertheless has the substantially flattened surfaces claimed in the '178 patent. This conclusion is wrong, for several reasons. First, as I detailed in my initial report through analytical geometry (i.e., the use of mathematics to accurately describe shapes), a flat surface is not curved, and the Pearl Plastic's finger grip is curved. The Court's construction of "substantially flattened

surfaces" covers surfaces "that are flat within a geometric, manufacturing tolerance" (emphasis added). Messrs. Turchi and Hutchison have applied an industry guideline for a flatness tolerance band to my measurements of the curved finger grip surface of the Pearl Plastic and, on this basis, have asserted that the Pearl Plastic literally infringes the '178 patent. I find this reasoning and this result to be incorrect because, as Messrs. Turchi and Hutchison recognize, the finger grip surfaces extend beyond these tolerance bands.

Second, Messrs. Turchi and Hutchison use the industry guideline for flatness tolerance appended to their opinions without regard to the design of the Pearl Plastic. The tolerance upon which they rely is not designed as a measurement of the flatness of a surface designed to be curved. Rather, it is intended to accommodate defects; particularly ones that are the consequence of material shrinkage. In several respects, the Pearl Plastic reflects industry design practice for avoiding such defects. These points are shown by the reference work cited by Messrs. Turchi and Hutchison and are reinforced by other references from the same source.

Third, the curvature present in the Pearl Plastic's finger grip is distinguishable from a surface that is designed to be flat but which, due to the imprecisions and inaccuracies of manufacturing, is in fact curved and lying within the tolerance limits. A curved surface falling within that tolerance level would have to have a radius of curvature nearly six times greater (and would thus form a much shallower arc) than the actual radius of curvature of the Pearl Plastic finger grip.

Fourth, the test proposed by Messrs. Turchi and Hutchison does not distinguish "substantially flattened surfaces" from surfaces that the '178 patent concedes are not "substantially flattened."

I explain these points in more detail below. Some of the terms I use are defined in Attachment A ("Definition of Terms"). Some of the equations I use are given in Attachment B ("Relevant Equations").

I. MESSRS. TURCHI AND HUTCHISON IMPROPERLY FOCUS ON ONLY A PORTION OF THE PEARL FINGER GRIP SURFACE IN THEIR INFRINGEMENT ANALYSIS

The P&G drawings 4360-01-A and 4359-01-A can be used to define the extent of the finger grip surfaces. I described the Pearl Plastic cross-section designs in my first report. In the case of the 14 mm applicator, the cross section is described by one radius of curvature, major and minor dimensions, and draft angles. Messrs. Turchi and Hutchison have not disputed this explanation. By examining the drawing of the 14 mm applicator, we find that that the finger grip surface has a chord length[1] of approximately 9.61 mm, at a minimum, which is the distance between the blending points on the finger grip surface (the points where the arcs that make up the gripping surfaces meet the arcs forming the rest of the rear surface of the Pearl's barrel). A construction showing the determination of this value based on scaled measurement from the drawing is shown in Attachment C ("14 mm construction"). The drawings do not show a flatness tolerance. Indeed, I do not expect them to have one because the Pearl Plastic finger grips are curved.

Mr. Turchi, Mr. Hutchison, and I agree on the definition of flatness tolerance. In my original report (p. 4), I cited the following definition of flatness tolerance:

> "[The] [f]latness tolerance specifies the tolerance zone bordered by two parallel planes within which the *entire surface* must lie." (Emphasis added.)

[1] A chord is a line segment with endpoints that lie on a circle. See Attachment A.

Mr. Turchi also cited this definition in his supplemental report (p. 2). At his deposition, Mr. Turchi was asked whether he agreed with this definition. His response (Depo., p. 414) was, "I do." Mr. Hutchison responded similarly (p. 206).

In addition, Mr. Turchi has stated that the Pearl Plastic finger grip surface extends beyond the flatness tolerance band he applies to the curved Pearl Plastic finger grip surface. In Mr. Turchi's supplemental report (p. 3), he writes, "Therefore, the Pearl Plastic literally infringes Claim 1 of the '178 patent because Dr. Moller's measured coordinate locations for the finger grip show that both sides of the finger grip have a significant flat *portion*" (emphasis added). Further, at his deposition, when presented with Exhibit 133, Mr. Turchi stated (p. 436):

> "The area that lies between points A and B would be considered under the way the court has construed flatness, it would be considered flat."

When asked the question, "As the surface continues beyond the A and B points, it falls outside of the flatness tolerance you set forth in your report?" he responded (p. 438), "I see. Yes, it does."

At his deposition, Mr. Hutchison also recognized that the Pearl Plastic finger grip surface extends beyond the flatness tolerance band he applies. In response to the question, "So what we have here is that a portion of the Pearl's finger grip would fall within the range of tolerance that you figured using Professor Moller's measurements?" he stated (p. 229), "That's correct."

The reasoning of Messrs. Turchi and Hutchison is inconsistent. On one hand they agree with my definition of flatness tolerance, but on the other hand they find that a portion of the Pearl Plastic finger grip surface falls within the flatness band they created. The Pearl Plastic

therefore *fails* their test. They have therefore erroneously concluded that the Pearl Plastic literally infringes the '178 patent.

II. MESSRS. TURCHI AND HUTCHISON USE THE SPI "FLATNESS TOLERANCE" INAPPROPRIATELY

Messrs. Turchi and Hutchison are inappropriately using the flatness tolerance value to conclude that the curved surface of the Pearl Plastic finger grip literally infringes the '178 patent. The flatness tolerance guideline in the SPI (Society of the Plastics Industry) reference cited by Messrs. Turchi and Hutchison is not intended to measure an acceptable amount of designed curvature but rather is intended principally to accommodate defects that are the consequence of shrinkage of the polymer material used to make the part. When a flatness tolerance is applied, defects which are of principal concern are sink marks and warpage. Sink marks are localized depressions in the molded part surface that are not reflected in the corresponding mold surface. Warpage is dimensional distortion in a plastic object after molding (Plastics Engineering Handbook, p. 334; relevant excerpts attached as Attachment D).

The behavior of polymer materials depends particularly upon temperature, flow, and pressure along with the respective histories of these quantities. When a polymer material is injected into a mold, it is hot and molten. In order for the material flow into the mold, pressure must be applied at the gate (i.e., the location where the polymer enters the mold). As the polymer fills the mold, the pressure is highest at the gate. During filling, the polymer molecules can become uncoiled and oriented in the direction of the material flow. This is known as orientation. As the material cools, it shrinks. Regions of the part that cool slowly tend to shrink more than regions that cool rapidly. This is because longer cooling times allow the polymer molecules to contract into more stable, compact configurations. When a part has regions that

cool at different rates, the amount of shrinkage will not be uniform. It is also the case that pressurizing the polymer as it cools will tend to reduce shrinkage. When shrinkage is not uniform, this can present itself as warpage or sink mark defects.

The Pearl Plastic has several features that reflect good design practice to avoid or reduce such defects. There is an integral cap on the end of the rearward portion of the barrel. Its presence tends to constrain the barrel wall in its as-molded shape as it cools. The gate is located on the rearward portion of the barrel at the 12 o'clock location and near the rear end of the barrel. The material thus enters the mold and forms the opposing grip surfaces essentially simultaneously. The filling and, very likely, the cooling patterns would be symmetric about a lengthwise plane passing through the 6 and 12 o'clock locations. Therefore, asymmetries among the 3 and 9 o'clock sides of the section would not be expected to occur. The principal flow direction is along the length of the part. This means any shrink due to molecule orientation that might be present would be principally along the length of the part. This would tend to not present itself as an alteration of the barrel section shape. Due to the gate location, the pressure in the rear portion of the barrel will be the highest in the part, thus reducing shrink in this region. The gate also adjoins one of the ribs in the cross section. Since the rib is thicker than the adjacent barrel wall, this gate placement is appropriate in that thicker regions need to be held at a higher pressure to reduce shrinkage.

The rearward portion of the barrel has a uniform wall thickness. The only exceptions are two ribs located furthest away from the grip surfaces (i.e., at the 12 and 6 o'clock positions). A closer examination of the flatness tolerance cited by Messrs. Turchi and Hutchison reveals that a uniform wall thickness is a principal way to address warpage. The footnote associated with flatness in that reference states, "Part design should maintain a wall thickness as

nearly constant as possible. Complete uniformity in this dimension is sometimes impossible to achieve. Walls of non-uniform thickness should be gradually blended from thick to thin." Further, the Plastics Engineering Handbook published by the SPI (p. 335) states:

> "Warpage caused by part design is the worst type, being nearly impossible to correct by molding conditions. For this reason, it is imperative that the part be designed to *prevent* objectionable warpage. As shrinkage is directly proportional to wall thickness, wall thickness is directly related to warpage. Hence wall thickness must be uniform to provide uniform shrinkage. Different wall thicknesses in the same part *must* result in either warpage, through stress-relief, or molded-in stress.... This varying wall thickness condition is probably the single largest cause of warpage."

When the Pearl Plastic design is taken into account, the flatness tolerance value used by Messrs. Turchi and Hutchison is inappropriately large. As shown by the design drawings attached to my initial report, the Pearl Plastic's curved finger grip surface is the result of design, not warpage. The results of their supplemental reports can in fact be used to demonstrate the high level of fidelity of the molded part to the design specification.[2] If an applicator had been designed to have flat grip surfaces, used the same material used in the Pearl Plastic, and had the same features of good design practice, it is highly unlikely that the surfaces of such a hypothetical object could warp to the extent allowed by the flatness tolerance.

---

[2] There is a high level of agreement between the Pearl Plastic as it is designed and the Pearl Plastic product. To reach this conclusion, I compared the empirical measurements that I made (which are also relied upon by Messrs. Turchi and Hutchison) with the dimensions given in the corresponding Pearl design drawing. Mr. Turchi found that the measured cross-section fell within the 0.610 mm tolerance band he applied over a distance of 7.27 mm on the 3 o'clock side and over 7.10 mm on the 9 o'clock side. If the same sort of tolerance band is placed on the design drawing, it can be seen that the inner edge of the band also describes the chord of a circle. This is shown in Attachment E. By using calculations from trigonometry, one finds the chord length to be 7.197 mm. This result differs by 0.10 mm from one of Mr. Turchi's findings and by 0.07 mm from the other. It differs from Mr. Hutchison's values by 0.101 mm, since Mr. Hutchison arrived at a value of approximately 7.096 mm for both the 3 o'clock and 9 o' clock sides.

III. THE PEARL PLASTIC APPLICATOR IS NOT FLAT WITHIN A GEOMETRIC MANUFACTURING TOLERANCE

A more appropriate way to estimate the proper tolerances to determine whether the Pearl Plastic has substantially flat grip surfaces is to use the chart for dimensions A, B, and C found in the reference attached as Exhibit A to Playtex's supplemental expert reports. While the figure in the reference shows an axisymmetric[4] part, the SPI has made clear that the tolerances and dimensions can be applied to other cross-sectional shapes as well. The introduction to the Standards for Molding Tolerances found in Chapter 28 of the Plastics Engineering Handbook (published by SPI) states:

> "By referring to the hypothetical molded article and its cross-section illustrated in the table, and by then using the applicable code number (e.g., A represents the diameter) in the first column of the table and the exact dimensions as indicated in the second column, readers can find the recommended tolerances either in the chart at the top of the table or in the two columns underneath. (Note that the typical article shown in cross section in the tables may be of round or rectangular or other shapes. Thus, dimensions A and B may be diameters or lengths.)" [Emphasis added.]

The chart for dimensions A, B, and C, therefore, can be used to estimate tolerances for both the Pearl Plastic finger grip cross-section and a cross-section that is designed to the Pearl Plastic's dimensions but which has flat finger grip surfaces like the applicator claimed and shown in the '178 patent. If the Pearl Plastic's curved finger grip surface cannot be contained within the tolerance band for the corresponding flat surface, then the Pearl Plastic does not infringe the '178 patent.

---

[4] "Axisymmetric" means: Having symmetry around an axis: *an axisymmetric cone* (American Heritage Dictionary, 2000).

I will now proceed to apply the tolerance for dimension "A" on the chart to the Pearl Plastic cross-section. The arcs corresponding to the gripping surfaces are specified to have a 10.92 mm radius. It is a principle of geometry that radius is one half of diameter. From the chart attached as Exhibit A to Playtex's supplemental expert reports we see that a diameter of 21.84 mm has tolerances of ± 0.12 mm (fine) and ± 0.20 (commercial). The tolerance on the radius is then one-half of those numbers: ± 0.06 mm (fine) and ± 0.10 (commercial). For fine tolerance, the radius can then range from 10.84 to 10.98 mm. For a commercial tolerance, the radius can range from 10.82 mm to 11.02 mm.

Next, I will construct a cross-section that is designed to the Pearl Plastic's dimensions but which has flat surfaces like the applicator claimed and shown in the '178 patent. In the 14 mm Pearl Plastic design, the crests of the gripping surfaces are 9.17 mm apart. As noted in Section I above, these surfaces meet the section ends. The distance between these junctures on the grip surface is approximately 9.61 mm. (See Attachment C). One can create a cross-section with perfectly flat surfaces that are 9.17 mm apart and 9.61 mm wide. From the row for dimension "A" on the Standards and Practices chart attached as Exhibit A to Playtex's supplemental expert reports, we see that the tolerances for the 9.17 mm dimension are ± 0.09 mm (fine) and ± 0.18 mm (commercial).

The actual molded surfaces can deviate from the perfectly flat surface shape in a variety of ways. One way in which the surfaces could deviate would be for them to become convex to the extent that they reach the limits indicated by the tolerance. This is shown in the figure that is Attachment F.

In the figure of Attachment F, the "Comparison Shapes" are the most tightly curved surfaces that can fall within the tolerance limits (recognizing again that my opinion is that one should not use a flatness tolerance to analyze a surface designed to be curved). The Comparison shapes, then, are two arcs of a circle. Their chord lengths are the length of the surface (at least 9.61 mm). Because the tolerance is applied to the distance between the surfaces, their heights are half the commercial tolerance range (half of 0.36 mm). The radius of these arcs can be calculated by basic trigonometry (see Attachment G). For the dimensions shown in the figure, the radius (i.e., the radius of curvature) of the "Comparison Shape" is 64.22 mm.

This radius is the smallest radius of curvature -- in other words, the deepest arc -- that the Comparison Shape surfaces can take on and still remain inside the commercial tolerance band. The Pearl Plastic's finger grip surfaces are not substantially flattened surfaces as that term has been construed by the Court. The smallest radius of curvature for the Comparison Shapes does not coincide with the range for the Pearl surfaces (10.92 ± 0.10 mm).[5] This radius is greater than the Pearl radius by a factor of nearly six. Messrs. Turchi and Hutchison have estimated that the finger grip surface is 12 mm wide. Turchi Depo., p. 426; Hutchison Depo., p. 229. If we were to use their value and again apply this test, the Pearl Plastic finger grip would differ from the Comparison Shapes by an even larger factor.

[5] The results are not significantly different for the 16 mm size of the Pearl.

IV. THE TEST PROPOSED BY MESSRS. TURCHI AND HUTCHISON DOES NOT DISTINGUISH "SUBSTANTIALLY FLATTENED SURFACES" FROM FLAT SURFACES THAT ARE ADMITTEDLY NOT "SUBSTANTIALLY FLATTENED"

The inappropriateness of the test created and applied by Messrs. Turchi and Hutchison can be further demonstrated by applying it to completely cylindrical surfaces having dimensions similar to those of the Pearl Plastic. Indeed, their test would find that a *portion* of the cylindrical surface lies within the tolerance range they construct. For the 14 and 16 mm Pearl Plastic applicators, the respective chord lengths are 5.72 and 6.13 mm, respectively.[6] If we apply the reasoning of Messrs. Turchi and Hutchison, these portions of the surface would be deemed "substantially flattened." Cylindrical grip surfaces, however, are excluded from the definition of "substantially flattened surfaces" by the '178 patent itself. If Playtex were to deem a segment of a perfectly cylindrical surface to be substantially flattened, then it will have erased any distinction between curved and flat surfaces. This is a nonsensical result under principles of analytic geometry, the language of the '178 patent, and the Court's construction of the term "substantially flattened surfaces."

If Playtex instead concedes such surfaces are not substantially flattened, then it will have conceded that the chosen tolerance is an inappropriate measure. There is no principled basis of which I am aware for claiming that chord lengths of 7.27, 7.10, and 7.096 mm[7] on the curved finger grip surface of the Pearl Plastic falling within the zone of tolerance result in "substantially flattened" surfaces yet chord lengths of approximately 5.72 mm and 6.13 mm would not result in "substantially flattened" surfaces.

---

[6] These chord lengths were calculated based on the equations found in Attachment B ("Relevant Equations").

[7] See Turchi, p. 3 and Hutchison, p. 2.

In his deposition, Mr. Turchi was presented with a copy of the Tampax '290 patent. When asked, "Did you go back and look at any prior art of the Tampax application or any other prior art to determine if the finger grips shown in that fell within the zone of tolerance you applied in your supplemental report?" he responded: "Based on the drawings, it would be hard to do that because all the drawings are not to scale and there are no dimensions indicated so that would be hard to see what the area would be. So to answer your question, I did not because I was not able to." Turchi Deposition, pp. 454-55.

I applied the tolerance band construction method of Messrs. Turchi and Hutchison to the '290 patent (Defendants' Exhibit 94-A). In the art shown in the '290 patent, the tubes include deformations that serve to positively locate the outer and inner tubes with respect to each other. The embodiment shows the preferred form of the complete applicator. Figure 3 in this patent shows a drawing of a transverse section view that includes grip surfaces. If we assume the drawing is to scale and that the tampon-carrying tube in the '290 patent was designed to be sized similarly to the Pearl Plastic (i.e., with a 14 mm barrel), the tolerance zone used by Messrs. Turchi and Hutchison can be constructed.[8] This construction is shown in Attachment H. The tolerance bands are constructed such that one side of each band is tangent to the deformations at their deepest extent. The construction reveals that the tolerance bands include a portion of the outer tube and entire deformation. The deformation includes an arc-shaped bottom and two sections which constitute the transition from the bottom to the outer tube surface. If we consider only the arc-shaped bottom that extends between the two transitional sections, we see that it, too, lies within the tolerance band. Under Messrs. Turchi and Hutchison's test, then, the bottom of this deformation found in the prior art is a "substantially flattened surface." It is my

[8] Mr. Turchi conceded at p. 455 of his deposition that the figures of the Tampax patent could be drawn to the scale of the Pearl.

understanding that this supplemental report will be provided to Mr. Butterworth, P&G's expert on invalidity.

DATE:____________________

____
James C. Moller, Ph.D., P.E.

ATTACHMENTS

A - Definitions of Terms
B - Relevant Equations
C - 14 mm construction
D - Excerpts from Plastics Engineering Handbook
E - Calculation of chord length using a 0.61 mm tolerance and a 10.92 mm radius
F - Diagram contrasting comparison shapes and perfectly flat surfaces
G - Calculation of radius of curvature using a 0.18 mm tolerance and a 9.61 mm surface length
H - Fig. 3 of Tampax patent with Turchi-Hutchison tolerance applied

understanding that this supplemental report will be provided to Mr. Butterworth, P&G's expert on invalidity.

DATE: 15 Aug 03

James C. Moller
James C. Moller, Ph.D., P.E.

ATTACHMENTS

A - Definitions of Terms
B - Relevant Equations
C - 14 mm construction
D - Excerpts from Plastics Engineering Handbook
E - Calculation of chord length using a 0.61 mm tolerance and a 10.92 mm radius
F - Diagram contrasting comparison shapes and perfectly flat surfaces
G - Calculation of radius of curvature using a 0.18 mm tolerance and a 9.61 mm surface length
H - Fig. 3 of Tampax patent with Turchi-Hutchison tolerance applied

A

How to Solve Word Problems in Geometry

Dawn B. Sova, Ph.D.

McGraw-Hill

New York San Francisco Washington, D.C. Auckland Bogotá
Caracas Lisbon London Madrid Mexico City Milan
Montreal New Delhi San Juan Singapore
Sydney Tokyo Toronto

Library of Congress Cataloging-in-Publication Data

Sova, Dawn B.
How to solve word problems in geometry / Dawn B. Sova.
p. cm.
Includes index.
ISBN 0-07-134652-X
1. Geometry—Study and teaching. 2. Word problems (Mathematics)
I. Title.
QA461.S68 1999
516—dc21 99-37603
CIP

McGraw-Hill
A Division of The McGraw-Hill Companies

1 2 3 4 5 6 7 8 9 0 DOC/DOC 9 0 9 8 7 6 5 4 3 2 1 0 9

ISBN 0-07-134652-X

The sponsoring editor for this book was Barbara Gilson, the editing supervisor was Donna Muscatello, and the production supervisor was Tina Cameron. It was set in Stone Serif by PRD Group.

Printed and bound by R. R. Donnelley & Sons Company.

This book is printed on recycled, acid-free paper containing a minimum of 50% recycled de-inked fiber.

Contents

Chapter 9

Circles

A *circle* is a set of points in a plane at a given distance from a given point in that plane, the *center,* and it is the source of numerous calculations in geometry. The shape lends itself to many real-world associations. From automobile wheels to Ferris wheels, the movement of circular objects provides an interesting source of speculation regarding size and distance problems. The addition of lines that form angles or function as *tangents,* lines in the plane of the circle that intersect with the circle in exactly one point, requires that you also call upon skills related to angles, lines in a plane, and triangles.

Definitions to Know

Central angle. An angle that has its vertex at the center of a circle and whose interior measure forms a **minor arc** of the circle while the remainder of the circle is defined as the **major arc,** as in Fig. 9-1.

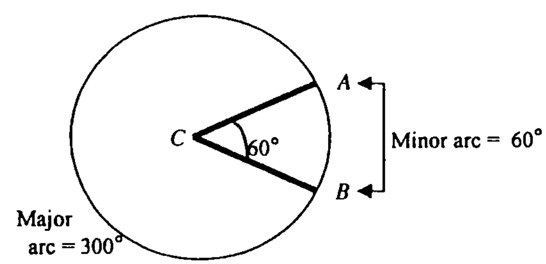


Fig. 9-1

Chord. A line segment with endpoints that lie on a circle, as in Fig. 9-2.

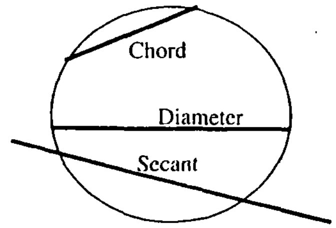


Fig. 9-2

Circumscribe. A circle is circumscribed around a polygon when all vertices of the polygon lie on the circle, as in Fig. 9-3.

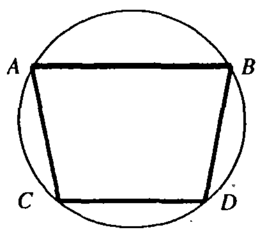


Fig. 9-3

Common tangent. A line that is tangent to two coplanar circles at different points and may be either internal, intersecting the segment that joins the centers, or external, not intersecting the segment that joins the centers, as in Fig. 9-4.

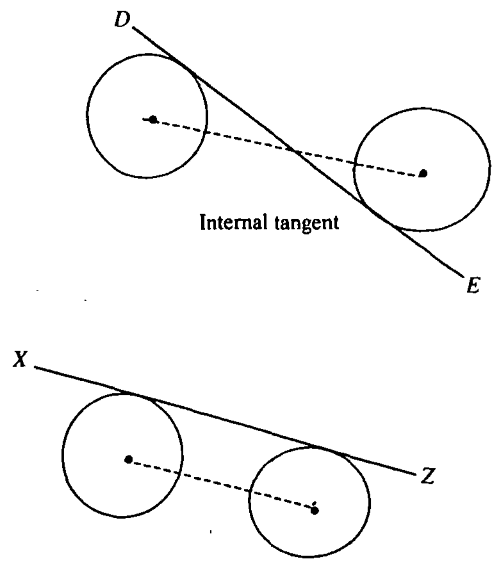


Fig. 9-4

Concentric circles. Circles that lie in the same plane and have the same center, as in Fig. 9-5.

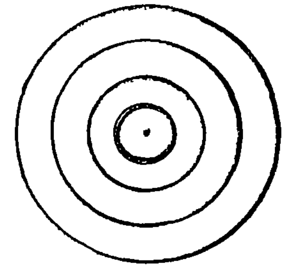


Fig. 9-5

Congruent arcs. Arcs that have equal measures.
Congruent circles. Circles that have congruent radii.
Diameter of the circle. A chord that contains the center of a circle, as in Fig. 9-2.
Inscribe. A polygon is inscribed in a circle when all vertices of the polygon lie on the circle, as in Fig. 9-3.
Inscribed angle. An angle with sides that are chords of the circle and whose vertex lies on the circle, as in Fig. 9-6.

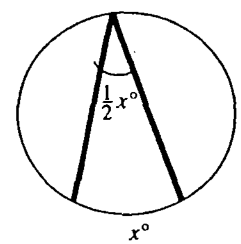


Fig. 9-6

Measure of a major arc. 360° minus the measure of the related minor arc, as in Fig. 9-1.
Measure of a minor arc. The same as the measure of its central angle, as in Fig. 9-1.
Point of tangency. The specific point where the circle and a tangent intersect, as in Fig. 9-7.

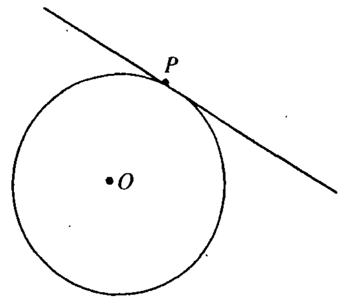


Fig. 9-7

Radius. Any line segment that joins the center of a circle to a point of the circle.
Secant. A line that connects the circle in two points and contains a chord, as in Fig. 9-2.
Semicircle. An arc with endpoints on a diameter of the circle, which measures 180°.
Tangent. The line in the plane of a circle that intersects the circle in exactly one point, as in Fig. 9-7.
Tangent circles. Coplanar circles that are tangent to the same line at the same point, as in Fig. 9-8.

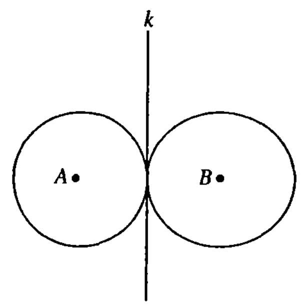


Fig. 9-8

Relevant Postulates and Theorems

Arc Addition Postulate

The measure of the arc formed by two adjacent arcs is the sum of the measures of these two arcs.

Theorem 1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency, as in Fig. 9-9.

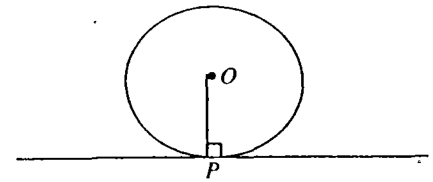


Fig. 9-9

Theorem 2

Tangents drawn to a circle from the same point are congruent, as in Fig. 9-10.

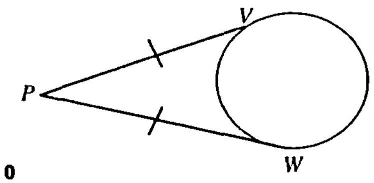


Fig. 9-10

Theorem 3

Two minor arcs are congruent if and only if their central angles are congruent.

Theorem 4

Congruent arcs have congruent chords, and congruent chords have congruent arcs.

Theorem 5

A diameter that is perpendicular to a chord bisects the chord and its arc, as in Fig. 9-11.

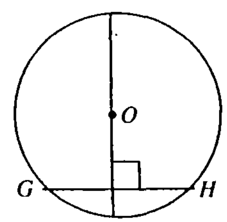


Fig. 9-11

Theorem 6

Chords that are equally distant from the center of a circle are congruent.

B

CRC

Standard Mathematical Tables

Sixteenth Edition

EDITOR
SAMUEL M. SELBY, PH.D.
Distinguished Professor of Mathematics,
Chairman, Mathematics Department
at University of Akron

THE CHEMICAL RUBBER CO.
18901 Cranwood Parkway, Cleveland, Ohio 44128

Library of Congress Card No. 30-4052

PREFACE

In order to meet the continually changing scientific environment which is for present in this generation, the policy of The Chemical Rubber Co. is to make a able the most effective and up-dated reference materials possible. The 15th edi resulted in bringing together a distinguished advisory group made up of specialist their respective areas of mathematics and science which produced an edition that has b accorded an excellent reception from its users. The sections involving mensuration, tr nometry, analytic geometry, curves and graphs, and the algebra of sets were comple rewritten and enhanced the worthiness of the contents in general. This new 16th edi follows the same successful format of authoritative, reliable, and quick table reference wl characterized the material of the 15th and all of its predecessor editions. This new 1 edition includes over 670 pages of important mathematical and scientific information wl has been examined and included in the light of importance in today's academic environm

The added content in statistics, together with additional sections covering determin and matrices as well as an extension to the octal decimal conversion table to now incl the hexadecimal and decimal conversion table, should make this 16th edition even m useful.

The improvement in this 16th edition of the *CRC Standard Mathematical Table* dictated only by the desire to make its need an important aid to the teaching professi the student of mathematics, physics or engineering, and to the many others who require mathematical fact or table for investigating and creating answers to the challenging pr lems in the present high school and college curricula.

Throughout the various tables where it is essential for their use, explanatory mate is included so that a fuller appreciation of same is available to the user. The 16th edit of the *CRC Standard Mathematical Tables* is patterned so as to give the necessary as to the many reputable teaching devices that are being marketed today to increase classro effectiveness with the hope that students will be stimulated to greater effort.

The editor wishes to use this means of expressing his sincerest appreciation to members of the advisory group who are listed in the forefront of this set of tables for tl continued cooperation, their excellent writing efforts, and for making his tasks easier perform.

It is hoped that the users of this 16th edition will continue to send in suggestions in the past. This important type of feedback has helped to guide the publication efforts The Chemical Rubber Co. to produce updated and important needs of reference r terial for the mathematics and scientific communities.

Samuel M. Selby, *Editor-in-Chief, Mathema*
Robert C. Weast, *Editor-in-Chief*

If s_k denotes the side of a regular polygon of k sides inscribed in a circle of radius R, then

$$s_{2n} = \sqrt{2R^2 - R\sqrt{4R^2 - s_n^2}}.$$

If S_k denotes the side of a regular polygon of k sides circumscribed about a circle of radius r, then

$$S_{2n} = \frac{2rS_n}{2r + \sqrt{4r^2 + S_n^2}}$$

If p_k and P_k denote, respectively, the perimeters of regular polygons of k sides inscribed in and circumscribed about the same circle, then

$$P_{2n} = \frac{2p_n P_n}{p_n + P_n} \quad \text{and} \quad p_{2n} = \sqrt{p_n P_{2n}}.$$

If a_k and A_k denote, respectively, the areas of regular polygons of k sides inscribed in and circumscribed about the same circle, then

$$a_{2n} = \sqrt{a_n A_n} \quad \text{and} \quad A_{2n} = \frac{2a_{2n}A_n}{a_{2n} + A_n}.$$

CIRCLES

In the following: R = radius, D = diameter, C = circumference, K = area.

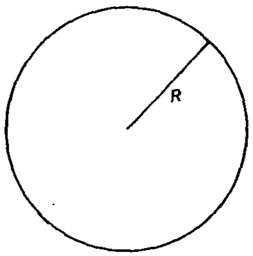
Circumference and Area of a Circle

$C = 2\pi R = \pi D \quad (\pi = 3.14159\cdots)$

$K = \pi R^2 = \frac{1}{4}\pi D^2 = 0.7854D^2$

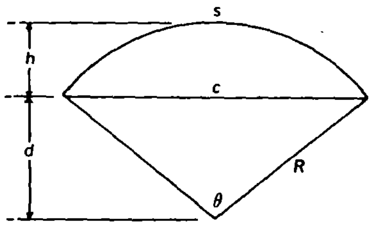
$C = 2\sqrt{\pi K} = \frac{2K}{R}$

$K = \frac{C^2}{4\pi} = \frac{1}{2}CR$



Sector and Segment of a Circle

Let the central angle θ be measured in radians ($\theta < \pi$).



$h = R - d, \quad d = R - h$

$s = R\theta$

$d = R\cos\frac{\theta}{2} = \frac{1}{2}c\cot\frac{\theta}{2}$

$= \frac{1}{2}\sqrt{4R^2 - c^2}$

$c = 2R\sin\frac{\theta}{2} = 2d\tan\frac{\theta}{2}$

$= 2\sqrt{R^2 - d^2} = \sqrt{4h(2R - h)}$

$\theta = \frac{s}{R} = 2\,\mathrm{Cos}^{-1}\frac{d}{R} = 2\,\mathrm{Tan}^{-1}\frac{c}{2d} = 2\,\mathrm{Sin}^{-1}\frac{c}{2R}$

$K\text{(sector)} = \frac{1}{2}Rs = \frac{1}{2}R^2\theta$

$K\text{(segment)} = \frac{1}{2}R^2(\theta - \sin\theta) = \frac{1}{2}(Rs - cd) = R^2\,\mathrm{Cos}^{-1}\frac{d}{R} - d\sqrt{R^2 - d^2}$

$= R^2\,\mathrm{Cos}^{-1}\frac{R - h}{R} - (R - h)\sqrt{2Rh - h^2}$

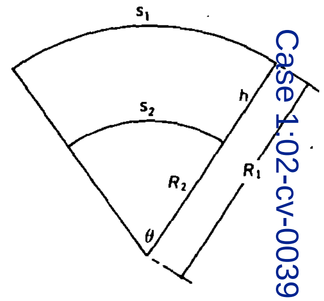
Sector of an Annulus

$h = R_1 - R_2$

$K = \frac{1}{2}\theta(R_1 + R_2)(R_1 - R_2)$

$= \frac{1}{2}\theta h(R_1 + R_2)$

$= \frac{1}{2}h(s_1 + s_2)$



CONIC SECTIONS

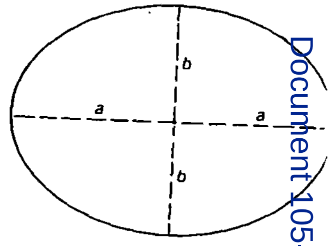
Ellipse

Let p = circumference, K = area

$p = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$ (approximately)

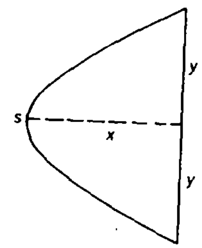
$= 4aE$ (exactly) See table of elliptic integral for E, using $k = \sqrt{a^2 - b^2}/a$.

$K = \pi ab$



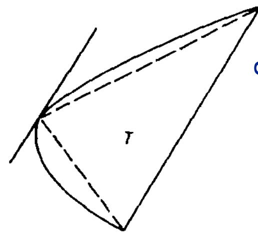
Parabolic Segment

$$s = \sqrt{4x^2 + y^2} + \frac{y^2}{2x}\log_e\left[\frac{2x + \sqrt{4x^2 + y^2}}{y}\right]$$



K (right segment) $= \frac{4}{3}xy$

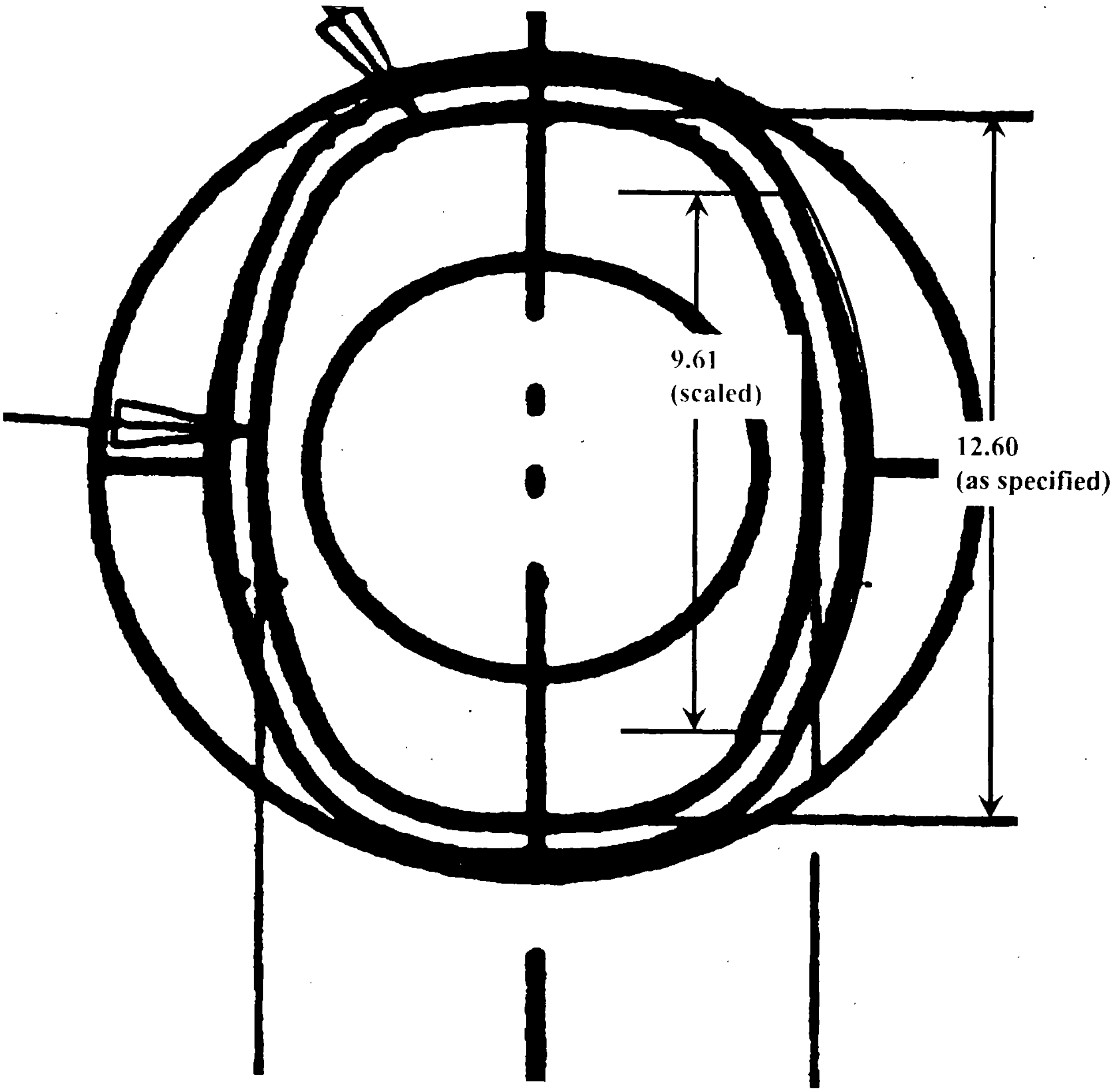
K (oblique segment) $= \frac{4}{3}T$, where T is the area of the triangle with base along the chord of the segment and with opposite vertex at the point on the parabola at which the tangent to the parabola is parallel to the chord of the segment.



CAVALIERI'S THEOREM FOR THE PLANE

If two planar areas are included between a pair of parallel lines, and if the two segments cut off by the areas on any line parallel to the including lines are equal in length, then the two planar areas are equal.

C


9.61
(scaled)
12.60
(as specified)

D

SPI Plastics Engineering Handbook

of the Society of the Plastics Industry, Inc.

Fifth Edition

Edited by

Michael L. Berins

CHAPMAN & HALL
New York • London

Library of Congress Catalog Card Number: 90-22784

This edition published by
Chapman & Hall
One Penn Plaza
New York, NY 10119

Published in Great Britain by
Chapman & Hall
2-6 Boundary Row
London SE1 8HN

Printed in the United States of America

15 14 13 12 11 10 9 8 7 6 5 4 3

Library of Congress Cataloging-in-Publication Data

Society of the Plastics Industry.
Plastics Engineering Handbook of the Society of the Plastics Industry/[edited by] Michael L. Berins.—5th ed.
p. cm.
Includes index.

1. Plastics—Handbooks, manuals, etc. I. Berins, Michael L.
II. Title.
TP1130.S58 1991
668.4—dc20 90-22784
CIP

British Library Cataloguing in Publication Data available

Please send your order for this or any **Chapman & Hall book to Chapman & Hall, 29 West 35th Street, New York, NY 10001, Attn: Customer Service Department.** You may also call our Order Department at 1-212-244-3336 or fax your purchase order to 1-800-248-4724.

For a complete listing of Chapman & Hall's titles, send your requests to **Chapman & Hall, Dept. BC, One Penn Plaza, New York, NY 10119.**

I am pleased to presen
in 1976, this version
numerous revisions an
At that last printing
duction would top 26 b
of this book was issue
billion pounds ann
Indeed, the story of
unparalleled innovatio
they also make it nece
information included i
Engineering Handboo

Fig. 11-43. ABS telephones typify high quality of surface finish obtainable with proper mold surfacing. (*Courtesy General Electric Plastics*)

gained by combining wide sweeping curves, domed surfaces, and a lustrous finish.

Molded Lettering

Names, monograms, dial numbers, instructional information, and the like, are frequently required on molded articles. The lettering must be applied in such a manner as not to complicate the removal of the article from the mold. This is accomplished by locating it perpendicular to the parting line and providing adequate draft.

While both raised and depressed letters are possible, the method to be used in constructing the mold will dictate which is the more economical.

When the mold is to be made by machining, raised letters on the molded piece will be less costly. A raised letter on the molded piece is formed by a corresponding depression in the mold, and it is far less costly to engrave or machine the letters into the face of the mold than it is to form a raised letter on the surface of the mold by cutting away the surrounding metal. On the other hand, if the mold is to be formed by hobbing, then the letters on the molded piece should be depressed, as it is the hob that must be machined. In making the hob, the letters are engraved into its surface. As the hob is sunk into the steel blank to produce the mold cavity, the letters are raised on the surface of the latter. These raised letters on the mold in turn produce depressed letters on the molded piece.

To improve legibility, depressed lettering, filled in with paint, sometimes is required. When hobbing of the whole mold is not practicable, the desired result generally can be accomplished by setting in a hobbed block carrying the lettering. When this insert is treated as a panel and the fin line is concealed by fluting, the appearance is not unpleasant.

The application shown in Fig. 11-44 is indicative of the type of precision lettering that can be accomplished in plastics products.

AVOIDING WARPAGE PROBLEMS*

Warpage is defined as "dimensional distortion in a plastic object after molding." It is directly related to material shrinkage; that is, as shrink-

*By Nelson C. Baldwin, Technical Service Engineer, Hoechst Celanese Corp.

Fig. 11-44. Molded-in letteri
(*Courtesy GE Plastics*)

age increases, the tenden
cur increases.

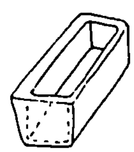
Anisotropic properties
some filled and reinforce
uid crystal polymers, a
warpage.

The purpose of this
guidelines to the designe
the factors that can caus
even shrinkage; namely,
sign, and molding condi

Part Design

Warpage caused by pa
type, being nearly imp
molding conditions. Fo
perative that the part be c
jectionable warpage. As
proportional to wall thic
is directly related to
thickness must be unifo
shrinkage. Different w
same part *must* result in e
stress-relief, or molded-
Examples 1 and 2 ill
nonuniform wall thickne

EXAMPLE 1:



Re

ɪer mold surfacing. (*Courtesy*

.s it is the hob that must
g the hob, the letters are
ɪce. As the hob is sunk
ɔroduce the mold cavity,
the surface of the latter.
the mold in turn produce
ɛ molded piece.
ɪty, depressed lettering,
sometimes is required.
whole mold is not prac-
ɪult generally can be ac-
in a hobbed block car-
ɪen this insert is treated
ine is concealed by flut-
not unpleasant.
wn in Fig. 11-44 is in-
precision lettering that
ɪ plastics products.

›AGE PROBLEMS*

"dimensional distortion
molding." It is directly
ɪkage; that is, as shrink-

Technical Service Engineer,

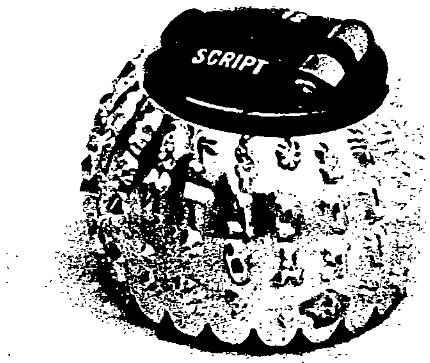


Fig. 11-44. Molded-in lettering in ABS typewriter head. (*Courtesy GE Plastics*)

age increases, the tendency for warpage to occur increases.

Anisotropic properties, such as are found in some filled and reinforced materials and in liquid crystal polymers, also can contribute to warpage.

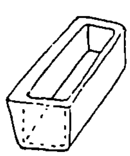
The purpose of this discussion is to offer guidelines to the designer and the processor on the factors that can cause warpage through uneven shrinkage; namely, part design, mold design, and molding conditions.

Part Design

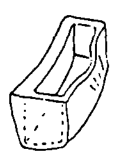
Warpage caused by part design is the worst type, being nearly impossible to correct by molding conditions. For this reason, it is imperative that the part be designed to *prevent* objectionable warpage. As shrinkage is directly proportional to wall thickness, wall thickness is directly related to warpage. Hence wall thickness must be uniform to provide uniform shrinkage. Different wall thicknesses in the same part *must* result in either warpage, through stress-relief, or molded-in stress.

Examples 1 and 2 illustrate warpage due to nonuniform wall thicknesses.

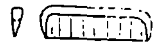
EXAMPLE 1:



Results in



EXAMPLE 2: Results in



This varying wall thickness condition is probably the single largest cause of warpage.

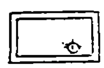
Another type of part design warpage concerns ribs and bosses. Indiscriminate location of ribs and improper selection of rib thickness can result in shrinkage patterns that will alter the shape of the entire molding.

Ribs should be no more than 50% of the adjacent wall thickness of the part to avoid sinks and possible distortion. However, ribs that are very thin compared to the main body can cause distortion due to different degrees of shrinkage.

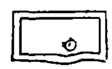
Bosses can affect the shape of the molded part if they are of a different wall thickness than the base to which they are attached, or if they are connected to a side wall of different thickness. Initial wall thickness for bosses should be the same as for ribs.

Example 3 depicts distortion from a rib tied to a side wall.

EXAMPLE 3:



Results in



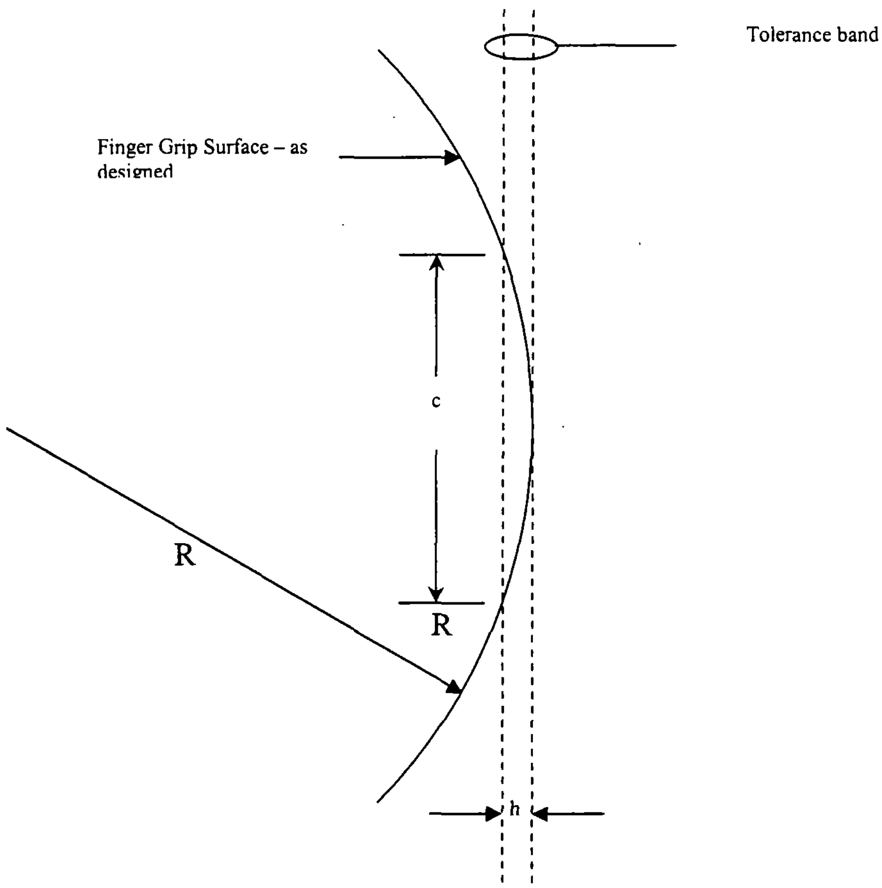
Ribs and bosses also can affect part geometry through changes in heat transfer from the mold. This subject will be covered later.

Mold Design

One of the most critical aspects of mold design, in unfilled and glass-filled crystalline polymers, is gate location. Its importance is due to many factors, including the inherent high shrinkage of the material and the anisotropic behavior it may exhibit.

Anisotropy refers to a shrinkage differential between the flow direction and the direction perpendicular (transverse) to flow. With unfilled materials, the greater shrinkage usually is encountered in the flow direction. Shrinkage in the transverse direction, conversely, usually ranges from 70 to 98% of the longitudinal shrinkage (direction of flow), depending on

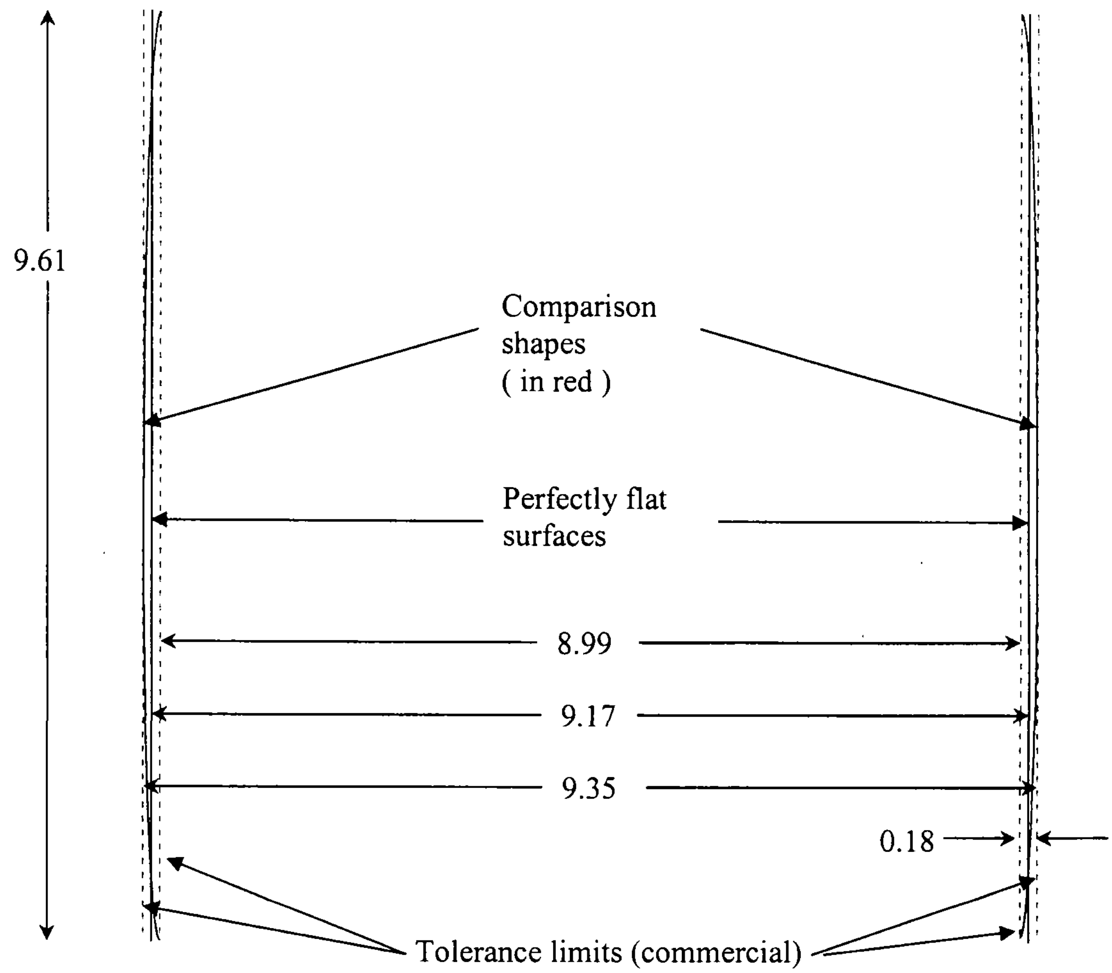
E



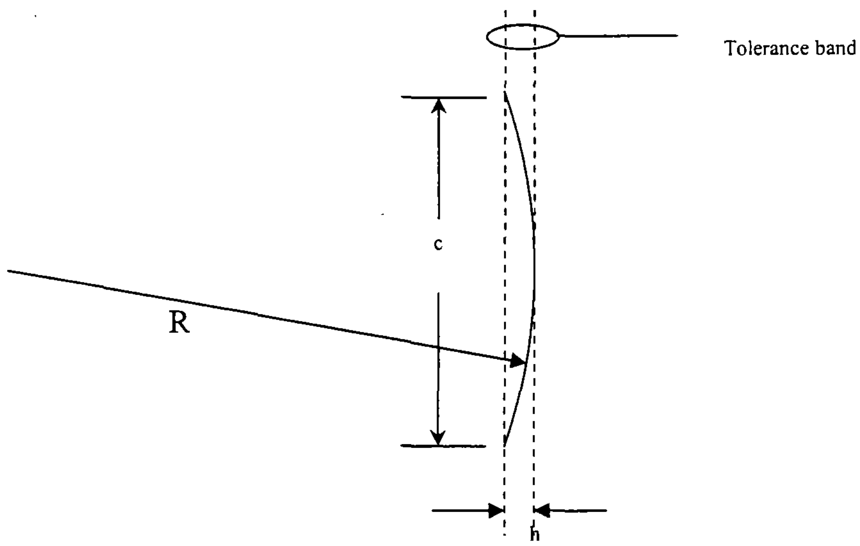
Given that R = 10.92 mm and h = 0.61 mm, calculate chord length c.

$$c=\sqrt{4h(2R-h)}=\sqrt{4\times 0.61(2\times 10.92-0.61)}=7.197\,mm$$

F


9.61
Comparison
shapes
(in red)
Perfectly flat
surfaces
8.99
9.17
9.35
0.18
Tolerance limits (commercial)

G



Given that c = 9.61 mm and h = (9.35 – 8.99)/2 mm = 0.18 mm, calculate radius R.

$$
\begin{aligned}
R &= \frac{\left(c^2 + 4h^2\right)}{8h} \\
&= \frac{\left(9.61^2 + 4 \times 0.18^2\right)}{8 \times 0.18} \\
&= 64.22\,mm
\end{aligned}
$$

H

Fig. 6.